

MATH 135 – QUIZ 1 – JAMES HOLLAND

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SOLUTIONS

Question 1. For what values of x is $|2x + 5| < 1$? Write your answer in interval notation.

Solution ∴

Note that $|2x + 5| < 1$ is equivalent to $-1 < 2x + 5 < 1$, which is the same as saying

$$-3 = \frac{-1 - 5}{2} < x < \frac{1 - 5}{2} = -2.$$

So the interval of possible x values is $(-3, -2)$.

Question 2.

- (i) What is the domain of the function $y = \log x$? Write your answer in interval notation.
 (ii) For what values of x does $\log(x) + \log(x) = \log(2x)$? Hint: remember part (i).

Solution ∴

- (i) The domain is $(0, \infty)$, since \log isn't defined at 0, nor at negative values.
 (ii) This is equivalent to $\log(x \cdot x) = \log(2x)$. This implies $x \cdot x = 2x$, or that $x \cdot (x - 2) = 0$, meaning x must be 0 or 2. Note that we used "implies" instead of "equivalent to": we must check that these values work.

$x = 0$ does not work, since 0 is not in the domain: $\log(0) + \log(0)$ has no meaning. $x = 2$ does work, because it *is* in the domain of \log . So $x = 2$ is the only value.

Question 3. Find the equation of a line passing through $(1, 2)$ that is perpendicular to the line given by

$$4x + 3(y + 3) = 0.$$

Write your answer in slope-intercept form.

Solution ∴

Slope intercept form means writing the line in the form $y = m \cdot x + b$ for some numbers m , the slope, and b , the y -intercept (since $x = 0$ implies $y = b$). First we will find the slope. Since it needs to be perpendicular to the given line, we must find the slope of the given line. To do this, we can just transform the given equation into point-slope formⁱ

$$(y + 3) = -\frac{4}{3}x.$$

So the slope is $-4/3$. Being perpendicular to this requires switching the change in x with the change in y , and also changing to the opposite direction: the slope of the perpendicular line is $-\frac{3}{-4} = 3/4$. Hence we know the line has the form $y = 3x/4 + b$. To find b , note that a line is completely determined by its slope and a single point, like the $(1, 2)$ given. So in particular, $2 = 3 \cdot 1/4 + b$ implies $b = 5/4$, and thus the equation of the perpendicular line is

$$y = \frac{3}{4}x + \frac{5}{4}.$$

Question 4. Consider the function f defined by

$$f(x) = \begin{cases} x^2 - 25 & \text{if } x > 4, \\ \sqrt{x} & \text{if } 1 \leq x \leq 4. \end{cases}$$

- (i) What is the domain of f ? Write your answer in interval notation.
 (ii) What is $f(4)$? What is $f(5)$?

Solution ∴

(i) f is defined only on $[1, \infty)$. This is because $x^2 - 25$ is defined for all $x > 4$, and \sqrt{x} is defined for all $1 \leq x \leq 4$.

(ii) Because $1 \leq 4 \leq 4$, we have $f(4) = \sqrt{4} = 2$. Because $5 > 4$, we have $f(5) = 5^2 - 25 = 0$.

ⁱor slope-intercept form. It doesn't matter, since we're only interested in finding the slope.